සියලු ම හිමිකම් ඇවිරිණි / (மුඟුට පුළිට්පු) නොපානා (All Rights Reserved)

අධ්යාපන අමාතකංශය අධ්යාපන අමාතකංශය අධ්යාපන අමාතකයා අධ්යාපය අධ්යාපය අමාතකංශය අධ්යාපන අමාතකංශය අධ්යාපන අමාතකංශය අ සහ්ඛ அமைச்சு සහ්ඛ அமைச்சு සහ්ඛ அமைச்சு සහ්ඛ **අත්යාපිත් නිර්ධාන අත්යාප** අමාතකංශය අධ්යාපන අමාතකංශය අධ්යාපන අමාතකංශය Ministry of Education Ministry of Education Ministry of E**පිහිර කිරීමට නැති කරනාදුන්** අධ්යාපන අමාතකංශය අවසාපන අමාතකංශය අධ්යාපන අමාතක් අධ්යාපන අමාතකංශය අධ්යාපන අමාතක් අධ්යාපන අමාතක් අධ්යාපන අමාතක අවසාපත අවසාපත අමාතක් අධ්යාපන අවාතක් අධ්යාපන අමාත්යාපන අමාතක් අධ්යාපන අවාත්යාපන අමාතක් අධ්යාපන අමාතක

G.C.E.(A.L) Support Seminar - 2023

குංයුක්ත ගණිතය II இணைந்த கணிதம் II Combined Mathematics II

10 E II

පැය තුනයි

மூன்று மணித்தியாலம் Three hours අමතර කියවීම් කාලය - මිනිත්තු 10 යි ගෙහනුස வாசிப்பு நேரம் - 10 நிமிடங்கள் Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number

Instructions:

* This question paper consists of two parts;

Part A (Questions 1-10) and Part B (Questions 11-17)

* Part A:

Answer all questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.

* Part B:

Answer five questions only. Write your answers on the sheets provided.

- * At the end of the time allotted, tie the answer scripts of the two parts together so that Part A is on top of Part B and hand them over to the supervisor.
- * You are permitted to remove only Part B of the question paper from the Examination Hall.
- * In this question paper, g denotes the acceleration due to gravity.

For Examiners' Use only

(10) Combined Mathematics II							
Part	Question No.	Marks					
	1						
	2						
	3						
	4						
A	5						
	6						
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(10) Combined Mathematics II

	Total
In Numbers	
In Words	

Code Numbers

Marking Examiner

Checked by:

2

Supervised by:

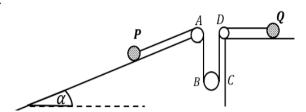
Part A

1.	Two particles A and B each of mass m are connected by a light inextensible	string. The pa	rticle A is given
	a velocity u along the direction of \overrightarrow{AB} . The coefficient of restitution	$\longrightarrow u$	
	between A and B is e. Find the velocities of A and B after the collision.	(m)	(m)
	Also find the impulsive tension in the string when it becomes taut.	A	В
			•••••
2.	Two equal particles P and Q are projected at the same instant respectively from	om points A an	d B at the same
2.	Two equal particles P and Q are projected at the same instant respectively from level on the ground, where $AB = a$. The particle P from A towards B with very level on the ground.	•	
2.		elocity <i>u</i> at 45	⁰ above AB and
2.	level on the ground. where $AB = a$. The particle P from A towards B with ve	elocity <i>u</i> at 45	⁰ above AB and
2.	level on the ground. where $AB = a$. The particle P from A towards B with vertice Q from B towards a with velocity v at 60° with respect to BA. If	elocity <i>u</i> at 45	⁰ above AB and
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2.	level on the ground. where $AB = a$. The particle P from A towards B with vertice Q from B towards a with velocity v at 60° with respect to BA. If	elocity <i>u</i> at 45	⁰ above AB and
2.	level on the ground. where $AB = a$. The particle P from A towards B with vertice Q from B towards a with velocity v at 60° with respect to BA. If	elocity <i>u</i> at 45	⁰ above AB and
2.	level on the ground. where $AB = a$. The particle P from A towards B with vertice Q from B towards a with velocity v at 60° with respect to BA. If	elocity <i>u</i> at 45	⁰ above AB and
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2.	level on the ground. where $AB = a$. The particle P from A towards B with vertice Q from B towards a with velocity v at 60° with respect to BA. If	elocity <i>u</i> at 45	⁰ above AB and
2.	level on the ground. where $AB = a$. The particle P from A towards B with vertice Q from B towards a with velocity v at 60° with respect to BA. If	elocity <i>u</i> at 45	⁰ above AB and

3. In the diagram is a light inextensible string attached to a particle of mass *m* placed on a fixed smooth plane inclined at to the horizontal. The string passes over a fixed small smooth pulley.

In the diagram PABCDQ is a light in inextensible string attached to a particle of mass m placed on a fixed smooth plane inclined at α to the horizontal. The string passes over a fixed small smooth pulley at A and under a smooth pulley of mass M and passes over a fixed small smooth pulley at B and attached to a particle Q of mass

m placed on fixed horizontal plane. PA is along a line of greatest slope AB and DC are vertical and DQ is horizontal. The system is released from rest with the string taut. Write down equations sufficient to determine tension in the string and accelerations of the particle and the movable pulley.



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4. A car of mass M kg is travelling along level road at constant velocity $u m s^{-1}$ with the engine working at maximum power of 3H W. Find the constant resistance force to motion of the car now the car. Now the car moves down a straight rode slope of 1 in 30 and the engine works at the same rate. If the resistance to motion is unchanged, show that maximum speed attains the car is $\frac{90 \, Hu}{90H-Mgu} m \, s^{-1} \left(H > \frac{mgu}{90}\right)$.

	A smooth hollow cylinder of internal radius a is fixed its axis vertical. One end	
	of a light inextensible string of length 2a is fixed to a centre of a point O on the	0
	lid of the cylinder. The other end of the string is attached to a particle of mass m	2a
	which moves in a horizontal code on the surface of the cylinder with constant	
	angular speed ω . (see in the figure)	⊗ n
	Show that the tension in the string is $\frac{2mg}{\sqrt{3}}$ and find the magnitude of the force	
	v-	
	exerted on the particle by the cylinder.	
υ.	In the usual notation ska $\underline{i} + \underline{j}$, $4\underline{i} + \underline{j}$ and $6\underline{i} + 3\underline{j}$ are the position vectors of point with respect to a point O. Also D be a point on AB such that	s A, B and C respectively,
	AD:DR=1:2 Find the nocition vector of the point D with respect to the point (
	AD:DB = 1:2. Find the position vector of the point D with respect to the point C	D. Show that the points
	O, D and C are collinear.	D. Show that the points
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7.	The figure shows a non-uniform rod AB of weight W, whose lower end A rests on rough horizontal ground. The rod is inclined at θ to the horizontal and rests in equilibrium attached by an inextensible string perpendicular to the rod AB at its
	centre of granting G, where AG: GB = 3: 1.
	Indicate clearly, what are the forces act on the rod AB. Given that reaction A is S
	and the string exerts a force P, then find the values of P and S in terms of W and θ
	In the diagram a light inextensible string attached to a uniform spere P of mass M placed on a fixed rough plank inclined at an angle θ to the
	horizontal. The string passes over a fixed small pulley at A and other end \downarrow_{mg}
	attached to a particle A of mass m. AQ is vertical and other part of the
	string is inclined at θ to the vertical.

(A/B)			12
Find,	(i)	$P(A \cap B), P(B \cap B)$	$\cap C$) and $P(A \cap B \cap C)$
	(ii)	$P(C \cap A)$ in form	k and the value of k
• • • • • • • • • • • • • • • • • • • •			
			1004, 1008, 1000, 1008, 996, 992, 1000, 1008, 1008, 1000 x. Find the mean and standard deviation of the values of x and deduce
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සංයුක්ත ගණිතය II இணைந்த கணிதம் II Combined Mathematics II

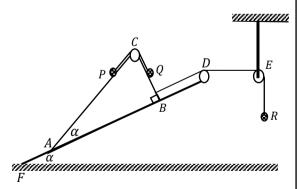


Part B

* Answer five questions only.

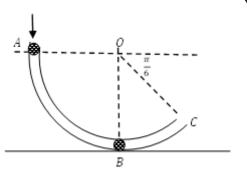
(In this question paper, g denotes the acceleration due to gravity.)

- 11. (a) A ball P attached to a balloon starts rising vertically upwards from rest with uniform acceleration λg from point O on the ground. When the balloon has reached a height h with the velocity v, the ball freely detached and it moves under gravity. When the balloon has reached further height h, another ball Q is projected vertically upwards from the same point O with the velocity v. It is given that the velocity of the ball is v when it is detached, while λ is a positive real constant.
 - (i) Draw the velocity- time graphs for the motion of each ball on the same diagram.
 - (ii) Show that $v = \sqrt{2\lambda gh}$ when it detached from the balloon.
 - (iii) Show that the time taken for the two balls to meet from the instant Q is projected is $\left(h + \frac{v^2}{2g}\right)$
 - (b) A ship A sailing with uniform velocity V in the direction of the South observes a boat P sailing the direction α degrees East of South. At the same instant, the ship A also observes another boat Q sailing the direction α degrees East of North.
 - (i) Draw in the same diagram, the velocity triangle for A and P and the velocity triangle for A and Q.
 - (ii) Find the speed of P and Q relative to A.
 - (iii) Given that initially the ship A is due East at a distance 24km from P and due North at am distance 6 km from Q, Find the distance between A and Q when the distance A and P is Least.
- **12.** (a) The triangles ABC in the figure is vertical cross sections through the center of gravity of a smooth uniform wedge of mass 3m with $A\hat{B}C = \frac{\pi}{2}$ and $B\hat{A}C = \alpha$. Such that the face AB placed on a line of greatest slope of a smooth fixed inclined plane. The inclined plane makes an angle α to the horizontal. The two ends of a light inextensible string passing over a small smooth small pulley fixed at C are



attached to the particles P and Q of masses 2m and m respectively. One end of another light inextensible string is attached, point of the wedge and passing over smooth small pulleys fixed at D and E. The other end is attached to the particle R of mass m. The string DE is in the horizontal position. At the initial position, the particles P and Q are held at AC and CB respectively such that, AF = a and the strings are taut. Here F is the point where the inclined plane meets the horizontal plane. The system is release from rest, assume that the wedge reaches the F before the particles P and Q reach foot of the wedge. Obtain the equations sufficient to determine the time taken by the wedge to reach the point F in terms of a and g.

(b) A thin smooth tube ABC is the shape of a circular arc of the radius a and centre O is fixed in a vertical plane with OA horizontal and the lowest point B of the tube touching a fixed horizontal floor as shown in the figure. The radius OC makes an angle $\frac{\pi}{6}$ with the downward horizontal.



Show that the radius to the composite body makes an angle

$$0 \le \alpha \le \frac{\pi}{3}$$
 with OB, the speed v of the composite body is given by $v^2 = 2ga\left(\frac{1}{(1+\lambda)^2} + \cos\theta - 1\right)$.

If $\lambda \leq \sqrt{2} - 1$, show that the composite body never leaves the tube and it oscillates along an arc of the tube.

Given that $\lambda = \sqrt{2} - 1$, Show that the reaction on the composite body from the tube is $\sqrt{\frac{3}{2}}$ mg when the body becomes instantaneously rest.

13. A thin light elastic spring, of natural length 3l, stands vertically with its lower end O fixed and carries a particle P of mass m fastened to its upper end. This particle is resting in equilibrium at the point A at a length 4l vertically above O, by a constant fixed force 3mg acts vertically upwards on the particle. Show that the modulus of elasticity of the spring is 6mg.

Now the particle is gently released from the point A show that the equation of the motion of the particle is $\ddot{x} + \frac{2g}{l}(x - \frac{5l}{2}) = 0$: where x is the displacement of the particle from O at time t for 3l < x < 4l. Rewrite the equation in the form $\ddot{X} + \omega^2 X = 0$ where $X = x - \frac{5l}{2}$ and $\omega^2 = \frac{2g}{l}$.

Assuming that $\dot{X}^2 = \omega^2(c^2 - X^2)$, find the amplitude C of this simple harmonic motion. When it is at point B at height 3l, vertically above point O. At the instant when the particle P is at B, another particle Q of mass m is gently collides and coalesces with P. Show that the composite body, will begin to move vertically downward with speed \sqrt{gl} .

Let, point D be the lowest point reached by the composite body, show that the motion of equation is $\ddot{y} + \frac{3g}{l} \left(y - \frac{8l}{3} \right) = 0$, where y is the verticle displacement of the particle from O, 2l < y < 3l.

Assuming that the solution of the above equation is of the form $y = \frac{8l}{3} + \alpha \cos \omega t + \beta \sin \omega t$, find the values of constants α, β and ω .

Hence, find the centre and amplitude of the simple harmonic motion performed by the composite body from B to D.

14. (a) The position vector of point A with respect to an origin O is $\sqrt{3}\underline{i} + \underline{j}$, Where \underline{i} and \underline{j} have the usual meaning. Let B the point such that OB=10 units and $B\widehat{O}A = 60^{\circ}$.

Take $\overrightarrow{OB} = \alpha \underline{i} + \beta \underline{j}$; $\alpha \neq 0$. Find \overrightarrow{OB}

- (i) Let C be the point on OB such that $\overrightarrow{AC} = \frac{\sqrt{3}}{2} \underline{i} \frac{5}{2} \underline{j}$. Find OC: CB Give that D is the midpoint of OA, find \overrightarrow{BD} .
- (ii) Let, E be the point such that $\overrightarrow{AE} = \frac{10}{17} \overrightarrow{AC}$ find \overrightarrow{BE} . Show that B, E and D point are collinear.

(b) ABCD is a trapezium in which the side AB is parallel to DC, AB is perpendicular to BD,

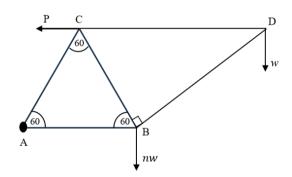
$$D\hat{A}B = 60^{\circ}$$
 and $C\hat{A}B = 30^{\circ}$.

Forces of magnitudes μp , 2p, $3\sqrt{3}p$ and λp act along AB, AD, CA and DB respectively, in the directions indicated by the order of the letters.

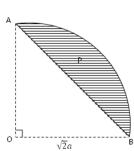
- (i) Show that the system is not in equilibrium any values of λ and μ
- (ii) If the resultant force of the system is along AD, then find λ and μ
- (iii) Now, a force αp and couple G acting in the same plane are added to the system in the direction through \overrightarrow{CD} . If the resultant force of the system is along DB, find the values of α and G.
- **15.** (a) The weights of the uniform rods AB, BC, CD and DA of equal lengths are 2W, 2W, 3W and W respectively. A quadrilateral frame is formed by joining rods freely at A, B, C and D. This is kept in the shape of a square by a light inextensible string connecting the centre of the gravity of the rods AB, BC and suspended freely from the joint A.

Show that the tension in the string is 9W. Also find, magnitude of reactions at the joints B and C for the rod BC.

(b) The framework shown in figure consists of five light rods smoothly joined their ends A, B, C and D. The rods AB, BC and AC are equal lengths, weighed *nw* and *w* are hung from B and D respectively. The framework smoothly hinged at A and kept in position of equilibrium by a horizontal force *P* applied at C with the rods ab and CD are in horizontal.

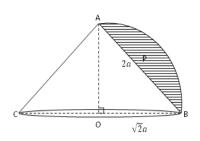


- (i) Show that $P = \left(\frac{2n+5}{\sqrt{3}}\right)w$.
- (ii) Draw a stress diagram. Using Bows notation and determine the stresses in the rods classifying them on tensions or thrust.
- (iii) Given that the maximum stress force in the rod BC is $10\sqrt{3}$ w, then show that $n \le 14$.
- **16.** (a) (i) Using integration show that the center of mass of a uniform circular sector of a circle of radius r, subtending at angle 2α at the centre is at a distance $\frac{2r\sin\alpha}{3\alpha}$ from the centre.
 - (ii) Show that centre of mass of a uniform hollow cone of base radius a and height h is at a distance $\frac{2}{3}h$ from its vertex.
 - (b) The figure shows the uniform lamina P obtained by removing an isosceles triangular lamina from a uniform circular sector of a circle of radius $\sqrt{2} a$ subtending at angle $\frac{\pi}{2}$ at its centre O. Show that the centre of mass is at a distance $\frac{4a}{3(\pi-2)}$ from O on its symmetric axis.



[See page ten]

(c) A composite body is formed by a hollow thin right circular cone of base diameter $\sqrt{2}a$, slant height 2a and the lamina P fixed along the slant height of the cone is shown in the figure. Given that mass of the cone is five forms of mass of the lamina. Then find the position of centre of mass of the composite body, taking OB and OA as X and Y axes respectively. If the composite body is suspended freely from the point A, show that the angle $tan^{-1}\left[\frac{5\pi-8}{2(9\pi-19)}\right]$ made by OA with the downward vertical.



17. (a) In analyzing of the records of a children's hospital revealed a few probability measures for the following events for male child chosen at random from th0se who are receiving treatment.

Event A: The child has Asthma.

Event *B*: The child has high blood pressure.

Event C: The child has diabetes.

It is given that the events A, B, C are mutually Independent and that,

$$P(B) = 0.3, P(A \cup B) = 0.37$$
 and $P(C) = 0.2$

- (i) Show that P(A) = 0.1
- (ii) Find P(B'/A'), where A' and B' denote the complements of A and B respectively.
- (iii) Find the probability that the child has diabetes but has neither high blood pressure nor asthma.
- (iv) Given that the child has just one of these ailments, find the probability that is asthma
- **(b)** The following table shows the distances to the nearest Kilometers travelled to work 120 employees of a company.

Distance	Number of Employees
0 -10	10
10-20	19
20-30	43
30-40	25
40-50	8
50-60	6
60-70	5
70-80	3
80-90	1

- (i) Using the transformation $y_i = \frac{1}{10}(x_i 45)$ estimated the mean and standard deviation of the distribution.
- (ii) The company decided to transfer the employees who travel more than 50 km, to closer branches of the company which are closer to them. Estimate the interquartile travel range of the distances travel to work by the remaining employees after the transfers are made.
