

9. Events A, B, C occur so that $P(A) = 8k$, $P(B) = 2k$ and $P(C) = k$; $k \neq 0$. Given also that $(A/B) = P(B/C) = 0$, $P(A/C) = P(A)$ and $(A \cup B \cup C) = \frac{11}{12}$,

- Find, (i) $P(A \cap B), P(B \cap C)$ and $P(A \cap B \cap C)$
(ii) $P(C \cap A)$ in forms of k and the value of k

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10. Each of the ten observations 1004, 1008, 1000, 1008, 996, 992, 1000, 1008, 1008, 1000 are expressed in the form $1000 - 4x$. Find the mean and standard deviation of the values of x and deduce the mean of the ten observations given.

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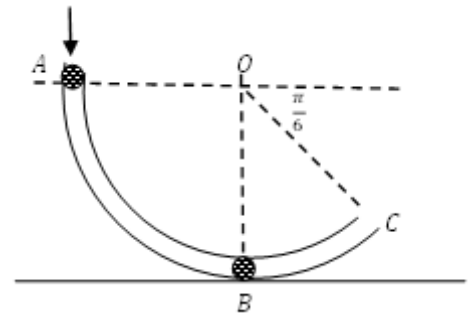
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- (b) A thin smooth tube ABC is the shape of a circular arc of the radius a and centre O is fixed in a vertical plane with OA horizontal and the lowest point B of the tube touching a fixed horizontal floor as shown in the figure. The radius OC makes an angle $\frac{\pi}{6}$ with the downward horizontal.



Show that the radius to the composite body makes an angle

$$0 \leq \alpha \leq \frac{\pi}{3} \text{ with } OB, \text{ the speed } v \text{ of the composite body is given by } v^2 = 2ga \left(\frac{1}{(1+\lambda)^2} + \cos\theta - 1 \right).$$

If $\lambda \leq \sqrt{2} - 1$, show that the composite body never leaves the tube and it oscillates along an arc of the tube.

Given that $\lambda = \sqrt{2} - 1$, Show that the reaction on the composite body from the tube is $\sqrt{\frac{3}{2}}mg$ when the body becomes instantaneously rest.

13. A thin light elastic spring, of natural length $3l$, stands vertically with its lower end O fixed and carries a particle P of mass m fastened to its upper end. This particle is resting in equilibrium at the point A at a length $4l$ vertically above O , by a constant fixed force $3mg$ acts vertically upwards on the particle. Show that the modulus of elasticity of the spring is $6mg$.

Now the particle is gently released from the point A show that the equation of the motion of the particle is $\ddot{x} + \frac{2g}{l}(x - \frac{5l}{2}) = 0$: where x is the displacement of the particle from O at time t for $3l < x < 4l$. Rewrite the equation in the form $\ddot{X} + \omega^2 X = 0$ where $X = x - \frac{5l}{2}$ and $\omega^2 = \frac{2g}{l}$.

Assuming that $\dot{X}^2 = \omega^2(c^2 - X^2)$, find the amplitude C of this simple harmonic motion. When it is at point B at height $3l$, vertically above point O . At the instant when the particle P is at B , another particle Q of mass m is gently collides and coalesces with P . Show that the composite body, will begin to move vertically downward with speed \sqrt{gl} .

Let, point D be the lowest point reached by the composite body, show that the motion of equation is

$$\ddot{y} + \frac{3g}{l} \left(y - \frac{8l}{3} \right) = 0, \text{ where } y \text{ is the vertical displacement of the particle from } O, 2l < y < 3l.$$

Assuming that the solution of the above equation is of the form $y = \frac{8l}{3} + \alpha \cos \omega t + \beta \sin \omega t$, find the values of constants α, β and ω .

Hence, find the centre and amplitude of the simple harmonic motion performed by the composite body from B to D .

14. (a) The position vector of point A with respect to an origin O is $\sqrt{3}\underline{i} + \underline{j}$, Where \underline{i} and \underline{j} have the usual meaning. Let B the point such that $OB=10$ units and $\hat{B}OA = 60^\circ$.

Take $\overrightarrow{OB} = \alpha\underline{i} + \beta\underline{j}$; $\alpha \neq 0$. Find \overrightarrow{OB}

- (i) Let C be the point on OB such that $\overrightarrow{AC} = \frac{\sqrt{3}}{2}\underline{i} - \frac{5}{2}\underline{j}$. Find $OC: CB$

Give that D is the midpoint of OA , find \overrightarrow{BD} .

- (ii) Let, E be the point such that $\overrightarrow{AE} = \frac{10}{17}\overrightarrow{AC}$ find \overrightarrow{BE} . Show that B, E and D point are collinear.

(b) ABCD is a trapezium in which the side AB is parallel to DC, AB is perpendicular to BD,

$$D\hat{A}B = 60^\circ \text{ and } C\hat{A}B = 30^\circ .$$

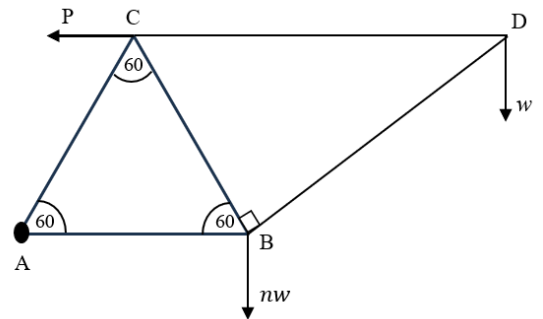
Forces of magnitudes μp , $2p$, $3\sqrt{3}p$ and λp act along AB, AD, CA and DB respectively, in the directions indicated by the order of the letters.

- Show that the system is not in equilibrium any values of λ and μ
- If the resultant force of the system is along AD, then find λ and μ
- Now, a force αp and couple G acting in the same plane are added to the system in the direction through \overline{CD} . If the resultant force of the system is along DB, find the values of α and G .

15. (a) The weights of the uniform rods AB, BC, CD and DA of equal lengths are $2W$, $2W$, $3W$ and W respectively. A quadrilateral frame is formed by joining rods freely at A, B, C and D. This is kept in the shape of a square by a light inextensible string connecting the centre of the gravity of the rods AB, BC and suspended freely from the joint A.

Show that the tension in the string is $9W$. Also find, magnitude of reactions at the joints B and C for the rod BC.

(b) The framework shown in figure consists of five light rods smoothly joined their ends A, B, C and D. The rods AB, BC and AC are equal lengths, weighed nw and w are hung from B and D respectively. The framework smoothly hinged at A and kept in position of equilibrium by a horizontal force P applied at C with the rods ab and CD are in horizontal.

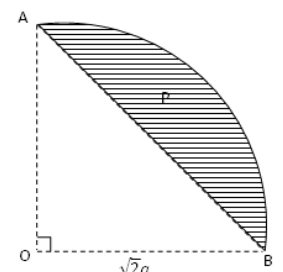


- Show that $P = \left(\frac{2n+5}{\sqrt{3}}\right)w$.
- Draw a stress diagram. Using Bows notation and determine the stresses in the rods classifying them on tensions or thrust.
- Given that the maximum stress force in the rod BC is $10\sqrt{3}w$, then show that $n \leq 14$.

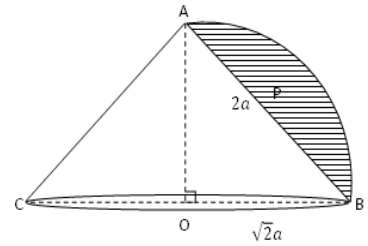
16. (a) (i) Using integration show that the center of mass of a uniform circular sector of a circle of radius r , subtending at angle 2α at the centre is at a distance $\frac{2rsin \alpha}{3 \alpha}$ from the centre.

(ii) Show that centre of mass of a uniform hollow cone of base radius a and height h is at a distance $\frac{2}{3}h$ from its vertex.

(b) The figure shows the uniform lamina P obtained by removing an isosceles triangular lamina from a uniform circular sector of a circle of radius $\sqrt{2}a$ subtending at angle $\frac{\pi}{2}$ at its centre O. Show that the centre of mass is at a distance $\frac{4a}{3(\pi-2)}$ from O on its symmetric axis.



- (c) A composite body is formed by a hollow thin right circular cone of base diameter $\sqrt{2}a$, slant height $2a$ and the lamina P fixed along the slant height of the cone is shown in the figure. Given that mass of the cone is five forms of mass of the lamina. Then find the position of centre of mass of the composite body, taking OB and OA as X and Y axes respectively. If the composite body is suspended freely from the point A, show that the angle $\tan^{-1} \left[\frac{5\pi-8}{2(9\pi-19)} \right]$ made by OA with the downward vertical.



17. (a) In analyzing of the records of a children's hospital revealed a few probability measures for the following events for male child chosen at random from those who are receiving treatment.

Event A: The child has Asthma.

Event B: The child has high blood pressure.

Event C: The child has diabetes.

It is given that the events A, B, C are mutually Independent and that,

$$P(B) = 0.3, P(A \cup B) = 0.37 \text{ and } P(C) = 0.2$$

- Show that $P(A) = 0.1$
- Find $P(B'/A')$, where A' and B' denote the complements of A and B respectively.
- Find the probability that the child has diabetes but has neither high blood pressure nor asthma.
- Given that the child has just one of these ailments, find the probability that is asthma

- (b) The following table shows the distances to the nearest Kilometers travelled to work 120 employees of a company.

Distance	Number of Employees
0 -10	10
10-20	19
20-30	43
30-40	25
40-50	8
50-60	6
60-70	5
70-80	3
80-90	1

- Using the transformation $y_i = \frac{1}{10}(x_i - 45)$ estimated the mean and standard deviation of the distribution.
- The company decided to transfer the employees who travel more than 50 km, to closer branches of the company which are closer to them. Estimate the interquartile travel range of the distances travel to work by the remaining employees after the transfers are made.