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கல்வி அமைச்சு
Ministry of Education

G.C.E.(A.L) Support Seminar - 2022

සංයුක්ත ගණිතය II
இணைந்த கணிதம் II
Combined Mathematics II

10 E II

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

Instructions:

* *This question paper consists of two parts;*
  Part A (Questions 1 – 10) *and* **Part B** (Questions 11 – 17)
* **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

* In this question paper, g denotes the acceleration due to gravity.

For Examiners' Use only

(10) Combined Mathematics II		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
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	16	
	17	
	Total	

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In Numbers	
In Words	

Code Numbers

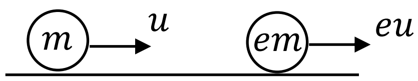
Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

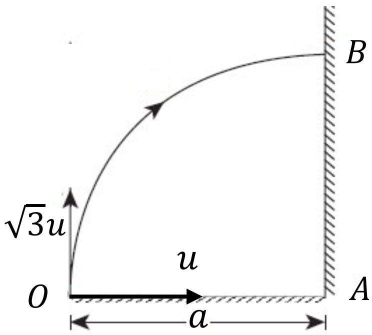
Part A

1. The coiefiecint of restitution between two particales A and B is e. The masses of A and B are m and λm respectively. Where $0 < e < 1$. The particals are moving with conatant speeds u and eu in the same horizontal line and in the same direction, as show in the diagram and collied directly.

Show that after the collision the speed of B is independent of e. Find the values of e for which the impulse excerted by B on A due to the collision has magnitude $\frac{6}{25}mu$



2. A particle is projected from a point O on a horizontal plane with the velocity whose horizontal and vertical components of u and $\sqrt{3}\,u$ respectively. The particle strikes the vertical wall all at the highest point B of its path and bouncing back to the plane OA. The wall is at a distance a from the point O as shown in the figure. Given that the coefficient of of restitution of the particle and the wall is $\frac{1}{2}$, Find

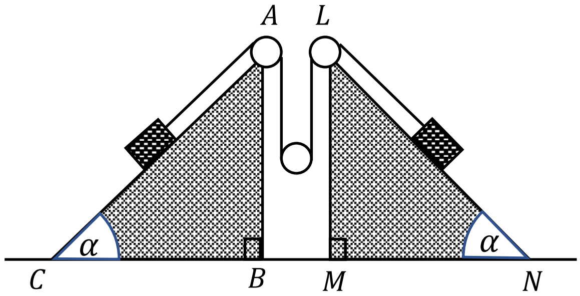


(i) The time taken by the particle to return to the horizontal plane OA

(ii) The distance from the point A to the particle where it reaches the plane OA

[see page three

3. The right-angle triangles ABC and LMN in the figure, are central vertical cross-sections through the Centre of gravity of two identical fixed smooth wedges, placed in a horizontal plane.
Where $A\hat{C}B = L\hat{N}M = \alpha$ and $A\hat{B}C = L\hat{M}N = \frac{\pi}{2}$. The line AC and LN are the lines of greatest slope of the relevant faces. Two particles P and Q of mass m_1 and m_2 are connected by a light inextensible string passing over to fixed pulleys at A and L, and passed under a smooth movable pulley of mass M. The system is released from rest. Obtain the equations sufficient to determine the tensions on the string and accelerations of the particles.



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4. A car of mass two metric ton is moving upwards a straight road of inclination $Sin^{-1}\frac{1}{10}$ to the horizontal at a steady speed of $32\ km\ h^{-1}$. If the resistance to motor is $400\ N$, find the power develop by the engine in kW.
Now the car moving in a straight horizontal road, given that the engine works in the same rate and resistance to motor is the same, find the acceleration of the car when the speed is $32\ km\ h^{-1}$.
$(g = 10\ m\ s^{-2})$

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[see page four

5. One end of a light inelastic string of length a is attached to a fixed point O and a particle of mass m is attached to the other end. The particle is held at the same level of O at a distance $\frac{a}{2}$ away from O and released. Find the speed of the particle just after the jerked of the string. In the subsequent motion when the particle is vertically below O by using conservation of energy, write an equation to determine the velocity of a particle.

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6. In the usual notation, let $x\underline{i} + y\underline{j}$ and $-2y\underline{i} + 2x\underline{j}$ are the position vectors of two points A and B respectively, with respect to a fixed origin O. Given that point C is the point on AB such that $AC:CB = 1:2$, find position vector of a point C, if the angle between $\overrightarrow{OC}$ and the $\overrightarrow{Oy}$ is 60^0, then show that $x^2 + y^2 + 4xy = 0$.

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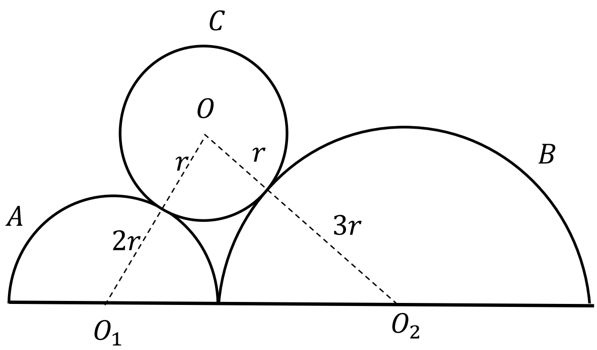
7. The figure shows A, B two smooth solid hemispheres of radii $2r$ and $3r$ respectively fixed on a horizontal plane, where O_1, O_2 are centers of A, B respectively. Another uniform Sphere C of radius r and mass m is gently placed on curved surfaces of $A,$ and B. Find the reactions on C which is exerted by $A\ and\ B$.



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8. A rod AB with one end A is contact with a rough horizontal plane, and the other end B is contact with a smooth vertical wall. The vertical plane through the rod AB is perpendicular to the wall. The coefficient of friction at point of contact A is μ and the centre of gravity of the rod AB divides $2:1$.Fine $tan\theta$ in the terms of μ when the rod is in limiting equation; when θ is the inclination of the rod to horizontal.

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9. Given that the two events satisfying the following conditions.

(i) The probability of only A occurring is 0.2

(ii) The probability of only B occurring is 0.1

(iii) The probability of neither A or B occurring is 0.6

Show that $P(A/B) = \frac{1}{2}$

10. The mean of ten numbers is 9.4. The deviations of the ten numbers from the real number k are given bellow.

d_i: -5, -2, -1, -1, -1, 0, 1, 1, 2, 6

Find the median and variance of the numbers.

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10 E II

Part B

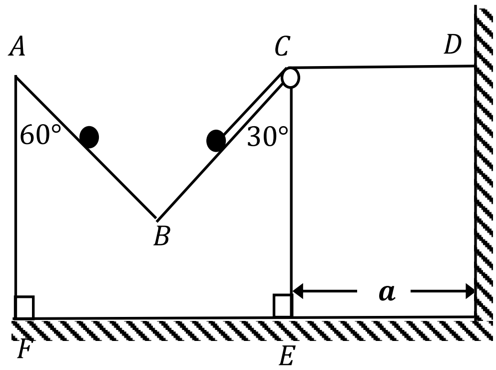
* Answer **five** questions only.

(In this question paper, g denotes the acceleration due to gravity.)

11. (a) A train P travelling in a straight path at a speed of $u\ km\ h^{-1}$ from a station A with uniform acceleration for half an hour and another one hour with a uniform retardation $f\ km\ h^{-2}$ and comes to rest at the station B. At that instant another train Q starting from rest, from the station B with same uniform acceleration of P for half an hour and acquire a maximum velocity. This maximum velocity is maintained and passing the station A. Sketch the velocity-time graph for the motions of P and Q in the same diagram. Hence find the acceleration of the train P and show that the time taken by the train Q, from B to A is $\frac{f}{f-u}$.

(b) A ship S is sailing due North with uniform speed $u\ km\ h^{-1}$ relative to earth. At a certain instant a boat A, at a distance $d\ km$ west of S and another boat B is at a distance $2d\ km$ east of S. The boat A and B travel in straight paths intending to intercept S with uniform speed $\frac{3u}{2}\ km\ h^{-1}$ and $2u\ km\ h^{-1}$ respectively, relative to earth.
By drawing velocity triangles for A and B, find which boat initially intercept the ship S.

12. (a) The $ABCEF$ is a vertical cross section through the center of gravity of a uniform smooth wedge of mass m. AB and CB are the lines of greatest slope of the respective faces with $F\hat{A}B = \frac{\pi}{3}$, $B\hat{C}E = \frac{\pi}{6}$ and $A\hat{F}E = C\hat{E}F = \frac{\pi}{2}$ such that the face containing FE placed on a smooth horizontal floor and the points A, C and D are in the same horizontal level as shown in the figure. The point D is a fixed point on the vertical wall which is at a distance a from the point C of the wedge.



One end of a light inextensible string passing over a small smooth pulley fixed at C is attached to a particle of mass $2m$ kept on CB and the other end is attached to the fixed-point D on the wall. A particle of mass m is held on AB. The system is released from rest as shown in the figure. Given that the wedge will reach the wall before either particle reaches to the point B. Obtain equations sufficient to determine the time taken by the wedge strikes the wall.

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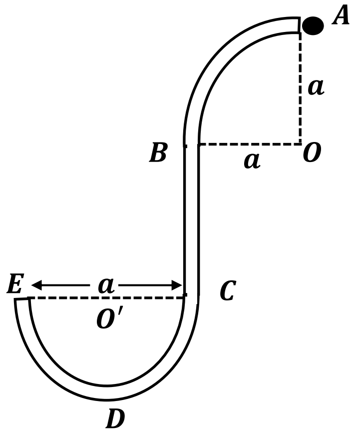
(*b*) A thin tube $ABCDE$ is fixed in a vertical plane as shown in the figure. The portion AB is a thin smooth circular tube with Centre O and radius a and arc AB strendy an angle $\frac{\pi}{2}$ at its Centre O. The portion CD is a thin vertical tube of length a. The portion CDE is a thin semicircular tube with Centre O' and radius $\frac{a}{2}$.



A particle P of mass m is placed inside the tube at A and gently released from rest.

(i) show that the speed V of the particle p when OP makes an angle $0 \leq \theta \leq \frac{\pi}{2}$ with OA is given by $v^2 = 2ga(1 - cos\theta)$ and find the magnitude of the reaction R on the particle from the tube in terms of θ and show that when $\theta = \cos^{-1}\left(\frac{2}{3}\right)$ the direction of the line of action of reaction is changed.

(ii) find the speed of the particle at the point E and further show that magnitude of the normal reaction is 8 mg at that point.

13. On a horizontal smooth table, six points $A,\ B, C, D, E\ and\ F$ lie on straight line such that $AB = BC = CD = DE = l\ and\ EF = 2l$. The points A and F are connected by a light elastic string of length $4l$. A smooth particle P of mass m is fastened to the point D on the string. The particle is pulled along the string on table to the point B and then gently released from rest. When the particle P is at a distance x from the point at time $t\ =\ t$ along AF, write down the equation of motion of particle P for $l \leq x \leq 2$ and show that in the usual notation, that $\ddot{x} + \frac{\lambda}{2ml}(x - 4l) = 0$, where λ is the modulus of elasticity of the string.

(i) By writing $X = x - 4l$ show that $\ddot{X} + \frac{\lambda}{2ml}X = 0$ assuming that the solution of the above equation is of the form $X = \alpha cos(\omega t) +\ \beta sin(\omega t)$

Find the constants $\alpha, \beta\ and\ \omega$. Hence, show that the particle passes the point C after time $\sqrt{\frac{2lm}{\lambda}}\cos^{-1}\left(\frac{2}{3}\right)$ with velocity $\sqrt{\frac{5\lambda l}{2m}}$.

(ii) Show that the equation of motion of the particle P by choosing Y for $2l \leq x \leq 4l$ can be written as $\ddot{Y} + \frac{\lambda}{ml}Y = 0$.

Assuming that the solutions of this equation is of the form,

$Y = \alpha'\cos(\omega'(t - t_0)) +\ \beta'\sin(\omega'(t - t_0))$ find the constants $\alpha',\ \beta'$ and ω'. Where

$$t_0 = \sqrt{\frac{2lm}{\lambda}}\cos^{-1}\left(\frac{2}{3}\right)$$

(iii) Show that the total time for the particle P to reach the point D is,

$$2\sqrt{\frac{l}{m}}\left\{\frac{\pi}{2} - \cos^{-1}\left(\frac{2}{7}\right) + \frac{1}{\sqrt{2}}\cos^{-1}\left(\frac{2}{3}\right)\right\}.$$

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14. (a) The position vectors of the points P and Q with respect to the point O are $\underline{p}$ *and* $\underline{q}$ respectively. The point L on OP is such that $OL: LP = 3: 4$ and the point N on OQ is such that $ON: NQ = 5: 2$. If M is the point of intersection of the straight lines PN and QL, show that $\overrightarrow{OM} = \underline{q} + \lambda(3\underline{p} - 7\underline{q})$. Here λ is a scaler. Obtain another expression for $\overrightarrow{OM}$ and find the position vector of M in terms of $\underline{p}$ *and* $\underline{q}$.

(b) A system consisting of three forces in the oxy plane act the points indicated below

Point	Position vector	Force
A	$3a\,\boldsymbol{i} + 2a\,\boldsymbol{j}$	$4P\,\boldsymbol{i} + 3P\,\boldsymbol{j}$
B	$-a\,\boldsymbol{i}$	$-P\,\boldsymbol{i} + 4P\,\boldsymbol{j}$
C	$-a\,\boldsymbol{j}$	$5P\,\boldsymbol{i} - P\,\boldsymbol{j}$

Here $\boldsymbol{i}$ *and* $\boldsymbol{j}$ denote unit vectors in the positive direction of coordinate axes ox and oy respectively and P and a are positive quantities measured in Newtons and meters respectively. Show that the system is equivalent to a single resultant force of magnitude $10P\ N$ and find direction and the equation of the line of action.
Also find the moment of the couple and its direction necessary to transfer the line of action to the line with equation $4y = 3x + 6a$

15. (a) Three uniform rods AB, BC and AC are smoothly jointed at their ends to from an equilateral triangle ABC. The rods AB and BC of equal weight W and the rod AC is of weight $2W$. The frame work ABC is freely suspended from vertex A.

Show that the inclination of AC to vertical is θ given by $tan\theta = \frac{\sqrt{3}}{4}$. Write down equations in terms of θ sufficient determine the reaction force at joint B for rod AB

(b) A frame work consists of light rods AB, BC, CD, DA *and* BD smoothly jointed at their ends as shown in the diagram. Here it is given that

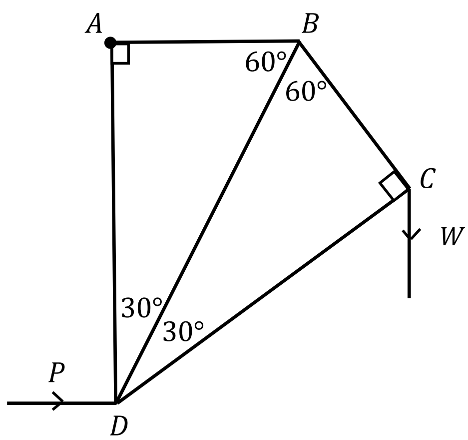
$AB = BC, AD = CD, A\hat{D}B = C\hat{D}B = 30°$ *and*
$A\hat{B}D = C\hat{B}D = 60°$

The framework is smoothly hinged at A and carries a weight of w at C. It is held equilibrium in a vertical plane, with AB horizontal and AD vertical by a horizontal force P applied at D. Draw a stress diagram, using Bow's notation, for the joints C, B, D.

Hence find

(i) The stresses in the five rods, stating whether, they are tensions or thrusts

(ii) The value of P and the reaction at A



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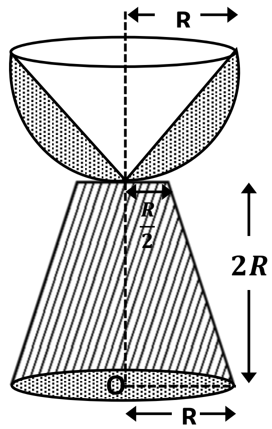
16. Show that the center of mass of

(i) a uniform solid hemisphere of radius a is at a distance $\frac{3}{8}a$ from its Centre

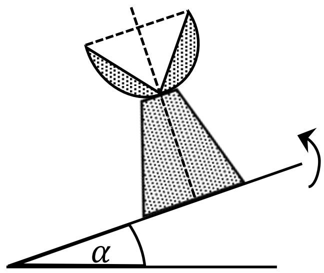
(ii) a uniform solid circular cone of height h is at a distance $\frac{1}{4}h$ from the centre of the base of the cone.

A uniform solid concreate flower Vass is made by rigidly fixing to a uniform solid concreate right circular frustum of radii of upper and lower circular bases $\frac{R}{2}$ and R respectively and height 2R is fastened at its upper circular face to the curved surface of the uniform concreate hemispherical shell, so that the two axes symmetry coincide as shown in the figure.



The hemispherical shell is made by removal of solid right circular cone of base radius R and height R from a uniform solid hemisphere of radius R. Bothe solid blocks are made are made of same material and mass per unit volume is ρ. Show that the distance from O to the Centre of mass of the flower Vass is $\frac{7R}{6}$.

The adjoining figure shows the vertical cross section of the above solid restore on a rough plane through in a line of greatest slope of the plane which is inclined to the horizontal. The inclination of the plane is increased, when the inclination of the plane with horizontal plane is α, show that if $\alpha < \tan^{-1}\left(\frac{6}{7}\right)$ and $\mu \geq \tan\alpha$, the flower Vass is in equilibrium. Where, μ is the coefficient of friction between lower face of the flower vass and the rough plane.



17. (a) Light bulbs manufactured by a company Consists of boxes A, B and C each containing three different standards of bulbs, in respective proportion $1:2:2$. Bulbs are identical in all respects except for their either defective or non-defective. The probability of bulbs of being damaged $0.00, 0.1$ and 0.2 in A, B and C respectively. A box is chosen at random and two bulbs are taken randomly and tested. Find the probability that,

(i) the bulbs are found to be defective

(ii) the box B was chosen, given that the tested bulbs to be non-defective

(b) The following table gives the class mark and corresponding frequency of a grouped frequency distribution of mark obtained by 70 students who passed the examination. In the examination the pass mark is 30

Class Mark	**frequency**
35	05
45	10
55	15
65	30
75	05
85	05

Using the transformation $y_i = \frac{1}{10}(x_i - 55)$, estimate the mean and the variance of the distribution of marks. The overall mean and the standard deviation of the marks of 100 students including 30 students who didn't pass are 48 and 21.5 respectively. Find mean and standard deviation of the marks of the 30 students who didn't pass.

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