

Test for selecting students to represent Sri Lanka at

International Mathematics & Science Olympiad Competition (Mathematics) - 2016

General Instructions	Index No: 2016/Oly/N/I/.....
This paper consists of two parts. <u>Answer all questions in both parts</u>	
Part A - 15 Short question answer questions (4 marks for each correct answer)	
<ul style="list-style-type: none"> • Answer should be written on the given dotted lines on paper itself. 	
Part B - 05 essay type questions (8 marks for each correct answer)	
<ul style="list-style-type: none"> • You should provide a descriptive answer step by step for each question and write on the papers provided. • No marks awarded for answers that are not clear. 	
Time – 2 hours	
The figures given may not be drawn to scale.	

Part I

(1) Which triangular number is 2016?

Answer:

(2) Now is year 2016. At year 2014, Mr. Perera celebrated his birth day which his age at that year was a square number and the sum of the digits of the year he was born also a square number. How old is he now?

Answer:

(3) Let a, b, and c be nonzero real numbers such that $a + \frac{1}{b} = 5, b + \frac{1}{c} = 12, c + \frac{1}{a} = 13$.

Find $abc + \frac{1}{abc}$.

Answer:

(4) Fill the empty square in the grid with an odd positive integer so that the sum (not necessary to be equal) of every numbers in columns and rows are a square of an odd positive two digit integer.

11		
		41
	33	

(5) If $h(a, b, c) = \frac{abc}{a+b+c}$, find $h(3\sqrt{5}, 6\sqrt{5}, 9\sqrt{5},)$

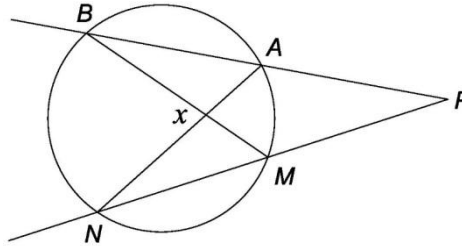
Answer:

- (6) In how many ways can put the symbol “+” or “−” in the brackets of the expression below so the result is a perfect square number?

$$1 () 2 () 3 () 4 () 5 () 6 () 7$$

Answer:

- (7) Point P lies outside a circle, and two rays are drawn from P that intersect the circle as shown. One ray intersects the circle at points A and B while the other ray intersects the circle at M and N. AN and MB intersect at X. Given that angle AXB measures 127° and the minor arc AM Measures (angle ANM) 14° , compute the measure of the angle at P.



Answer:

- (8) How many numbers are there that appear both in the arithmetic sequence 10, 16, 22, 28,1000 and the arithmetic sequence 10, 21, 32, 43,, 1000.

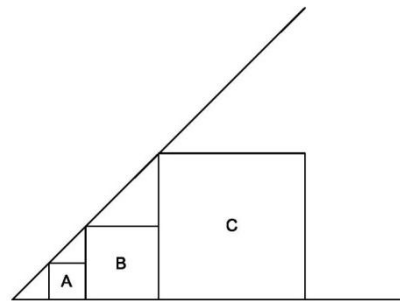
Answer:

- (9) The positive integers $v, w, x, y,$ and z satisfy the equation $v + \frac{1}{w + \frac{1}{x + \frac{1}{y + \frac{1}{z}}}} = \frac{222}{155}$

Compute, $10^4v + 10^3w + 10^2x + 10^1y + 10^0z$.

Answer:

- (10) Square A is adjacent to square B which is adjacent to square C. The three squares all have their bottom sides along a common horizontal line as shown. The upper left vertices of the three squares are collinear. If square A has area 24, and square B has area 36, find the area of square C.



Answer:

- (11) How many triangles can be formed such that its perimeter is 21cm. (The length of sides of the triangle are integer lengths)

Answer:

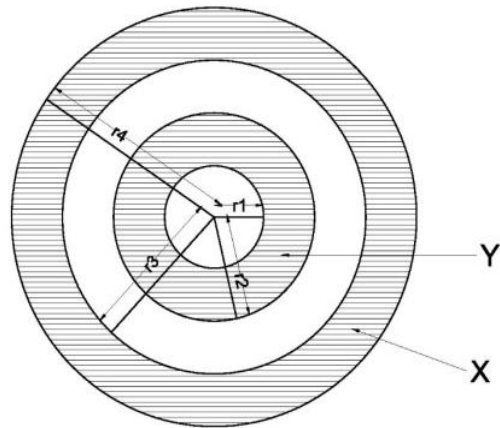
- (12) The product of two positive numbers is equal to 50 times their sum and 75 times their difference. Find their sum.

Answer:

- (13) Let N be the sum of the first four, three-digit prime numbers. Find the sum of the prime factors of $\frac{N}{2}$.

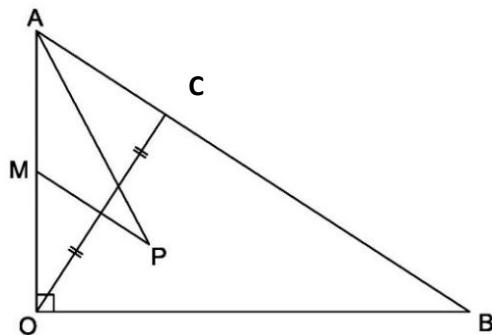
Answer:

- (14) 4 concentric circles are shown in the figure such the radii of the circles r_1, r_2, r_3 and r_4 in the ratio of 1:2:3:4 respectively. Find the ratio of area X:Y



Answer:

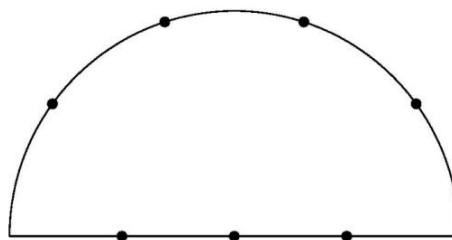
- (15) $\triangle AOB$ is a right angle triangle such that $\angle AOC : \angle OBA = \angle OBA : \angle COB = 1:2$. MP is perpendicular to OC and AP bisect the angle OAC . Find the magnitude of angle APM . (in degree)



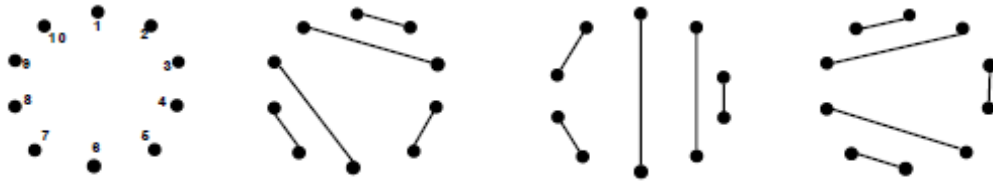
Answer:

Part II

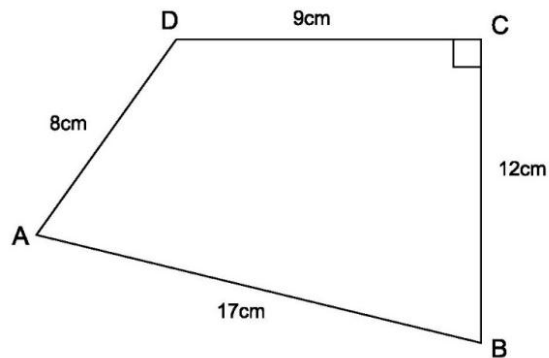
- (1) (a) There are 4 points on arc on the semi-circle and 3 points on its diameter. How many triangles can be formed by choosing any 3 points as their vertices.



- (b) Ten distinct points are placed in a circle. All ten of the points are paired so that the line segments connecting the pairs do not intersect. In how many different ways can this pairing be done? (Three examples are given)



- (2) (a) The work team was working at a rate fast enough to process 1250 items in ten hours. But after working for six hours, the team was given an additional 150 items to process. By what percent does the team need to increase its rate so that it can still complete its work within the ten hours?
- (b) The number of fishes in a tank “A” decreases by 120 per year. While the number of fishes in tank “B” increases by 80 per year. There were 12840 fishes in tank “A” in the year 2000 and 9700 fishes in tank B in year 2003. In what year will be the number of fishes in tank “B” start to exceed the number of fishes in tank “A”?
- (3) The quadrilateral ABCD is given bellow. Find the area of quadrilateral ABCD by showing steps.



- (4)(a) Find the sum of all four-digit integers whose digits are a rearrangement of the digits 1, 2, 3, 4 such as 1234, 1432, or 3124.
- (b) If x_1, x_2, x_3 are positive integers and $x_1+x_2+x_3=19$, $x_1+2x_2+x_3=36$, $x_1+x_2+2x_3=8$
Find the value of $x_1+x_2-x_3$
- (5) (a) A jar was filled with jelly beans so that 54% of the beans were red, 30% of the beans were green, and 16% of the beans were blue. Alan then removed the same number of red jelly beans and green jelly beans from the jar so that now 20% of the jelly beans in the jar are blue. What percent of the jelly beans in the jar are now red?
- (b) How many seven-digit positive integers do not either start or end with 7?